Possible Voter Control in k-Approval and k-Veto Under Partial Information

Gábor Erdélyi[1] Christian Reger[2]

Abstract: We study the complexity of possible constructive/destructive control by adding voters (PCCAV/PDCAV) and deleting voters (PCCDV/PDCDV) under nine different models of partial information for k-Approval and k-Veto. For the two destructive variants, we can further settle a polynomial-time result holding even for each scoring rule. Generally, in voter control, an external agent (called *the chair*) tries to change the outcome of the election by adding new voters to the election or by deleting voters from the election. Usually there is full information in voting theory, i.e., the chair knows the candidates, each voter's complete ranking about the candidates and the voting rule used. In this paper, we assume the chair to have partial information about the votes and ask if the chair can add (delete) some votes so that his preferred (despised) candidate is (not) a winner for at least one completion of the partial votes to complete votes.

Keywords: computational social choice, voting, control, algorithms, complexity, partial information

1 Introduction

For a long time voting has been used to aggregate individual preferences and has applications in informatics, politics or economy (e.g., design of recommender systems [Gh99] or machine learning [Xi13]). In computer science applications we are often dealing with huge data volumes and hence the numbers of candidates and voters may be exorbitantly large. Thus it makes sense to study the computational complexity of (decision) problems related to voting. Such problems are *winner*, which asks if a given candidate c is a winner of an election or not, *manipulation* and *bribery*. In manipulation a group of strategic voters try to coordinate their votes to make a candidate c win, whereas in bribery an external agent, called the *briber*, alters some votes to reach his aim. The groundbreaking papers of Bartholdi et al. [BTT89a, BTT89b, BTT92] suggest that computational hardness provides a (worst case) barrier against manipulative attacks. Unfortunately various voting problems tend to be easy for frequently used voting rules like scoring rules [Li11, BTT89a, FHH09]. On the other hand, a manipulative agent traditionally has full information in voting theory, i.e., he knows the preference orders of each voter about all candidates. In many real-world settings, however, this assumption is not realistic. There is already a bunch of literature dealing with problems under uncertainty.

In this paper, we analyze the complexity for constructive control by adding (CCAV) and deleting voters (CCDV) in k-Approval and k-Veto under partial information, as these families of voting rules belong to the most prominent voting rules. Moreover, we study destructive control by adding (DCAV) and deleting voters (DCDV) under partial information. In

[1] University of Siegen, erdelyi@wiwi.uni-siegen.de
[2] University of Siegen, reger@wiwi.uni-siegen.de

CCAV (DCAV), the chair tries to add a limited number of new voters to an election to make a given candidate c win (prevent c from winning). In CCDV (DCDV), the chair deletes some voters from the election in order to make c (not) a winner. Natural examples are political elections where the voting age is lowered or raised. For CCAV, three natural generalizations of full information arise. We study if it makes a difference if the previous (*registered*) voters or the new (*unregistered*) voters are partial (or both). On the one hand, it seems natural that the chair has (due to surveys or former elections) full (or at least enough) knowledge about the registered voters, but only partial knowledge about the unregistered voters. On the other hand, the chair may personally know the unregistered voters (at least some of them) or has a belief about them, but has only little knowledge about the registered voters. We especially regard an optimistic variant of control under partial information, i.e., we ask if c can be made a winner for *at least one* way to complete the partial votes to complete rankings in the resulting election.

Related Work First of all, CCAV and CCDV under *full* information were introduced by Bartholdi et al. [BTT92]. Hemaspaandra et al. [HHR07] extended this research for destructive control. Control results especially for k-Approval and k-Veto were published in [Li11]. The first work about voting under *partial* information is by Konczak and Lang [KL05]. They introduced the possible/necessary winner problems and allowed votes to be partial orders instead of linear orders as before in literature. They studied under which conditions a candidate is a winner for *at least one* (*possible winner*) or for *all* (*necessary winner*) ways to complete the partial votes to full rankings. Our problem is related to possible winner in a sense that we consider nine different ways to reflect partial information and partial orders is one of them, and we ask if the chair can make his favorite candidate a winner under *at least one* completion of the partial votes. Possible winner has also been studied in [XC11, BD10, Ch12]. Finally we mention [CWX11] where a manipulator has partial information in form of a general information set (instead of partial orders) containing all possible profiles that can be achieved by completing the partial profile. Basically, their model is a generalization of all the models considered by us. (Necessary and possible) bribery and (necessary) voter control under these models have recently been studied in [BER16, ER16, Re16]. In these papers, one could observe that PC (aka partial orders) often yields hardness results where other (very general) partial models produce P results for the same problem. This work thus often refers to some previous works about partial information. One goal of us is to perform an extensive complexity study for different structures of partial information and this paper is one important part of this study. Another aim of this paper is to check if the possible winner variant of control often increases the complexity compared to control under full information. On the one hand, possible winner is only easy for Veto and Plurality, but hard for k-Approval and k-Veto ($k \geq 2$) given partial orders [XC11, BD10]. Thus at most these two voting rules can yield P results at all for PCCAV and PCCDV as possible winner is a special case of possible control. (Notice that this is not true for PCCAV with complete registered voters (see Section 4 for further information) although we can actually reduce some hardness results *indirectly* from possible winner.). On the other hand, we know from [BER16] that (necessary) bribery often makes the complexity jump from P to hardness by combining necessary winner and bribery.

Organization This paper is organized as follows. In Section 2, we provide some preliminaries and briefly introduce nine models of partial information before we define our problems studied in this paper in Section 3. Finally, we give our complexity results for possible control in Section 4. In Section 5, we provide results on possible destructive control under partial information. Section 6 gives a short conclusion.

2 Preliminaries

An *election* is defined as a pair $E = (C,V)$ where C is a finite *candidate set* (with $|C| = m$) and V a finite *voter set*. Usually, each voter v_i is given as a strict linear order (i.e., total, transitive and asymmetric) $\succ_{v_i}$ over C representing his preferences, i.e., there is full information. In this paper, however, votes are often given partially in form of some model which is specified later. An *n-voter profile* $P := (v_1,\ldots,v_n)$ on C consists of n voters $v_1,\ldots,v_n$ given as strict linear orders or (for a *partial profile*) partial in terms of a certain model of partial information. A *completion* or *extension* of a partial profile P is a complete profile P' not contradicting P (in other words, each vote is completed in a way that its partial structure is preserved.) We also borrow the notion *information set* from game theory and say that $P' \in I(P)$ where $I(P)$ is the information set of P containing all complete profiles not contradicting P. A *voting rule* (more precisely, a *voting correspondence*) $\mathscr{E}$ maps an election $E = (C,V)$ to a subset of C which is called the *winner set*. As we use the *non-unique winner model*, we allow a voting rule to have exactly one, more than one or no winner at all. In this paper, we restrict ourselves to *scoring rules* defined by a vector $\alpha := (\alpha_1,\ldots,\alpha_m)$ with non-negative and non-increasing entries. Each voter assigns α_1 points to his favorite candidate, α_2 points to his second most preferred candidate and so on. The candidate(s) with the highest score are the winner(s). Important special cases are k-Approval and k-Veto. In k-Approval each voter assigns one point to his k most favorite candidates and zero points to the remaining candidates, whereas in k-Veto each voter gives zero points to his k least favorite candidates and one point to the remaining ones. 1-Approval is also known as Plurality, we further say Veto instead of 1-Veto. Note that for a fixed number m of candidates, k-Approval equals $(m-k)$-Veto. In our analysis, however, only k is fixed, but m is variable.

We regard the following nine models of partial information which can be found in [BER16] and the references therein. Besides, we briefly point out their interrelations.

Gaps (GAPS) [BER16] and One Gap (1GAP) [Ba12]. For each vote v, there is a partition $C = C_1^v \dot\cup \ldots \dot\cup C_{2m+1}^v$, a total order for each C_k^v (k even) and no information at all for odd k. Note that possibly $C_k^v = \emptyset$ for some k. We further have $c_i \succ c_j \; \forall c_i \in C_i^v, c_j \in C_j^v$ with $i < j$. A special case is 1GAP, where in each vote some candidates are ranked at the top and at the bottom of the votes, and there is at most one gap. Formally, 1GAP is a special case of GAPS with $C_k^v = \emptyset$ for each $k \in \{1,5,6,\ldots,2m+1\}$ and each voter v.

Top-/Bottom-Truncated Orders (TTO,BTO) [Ba12]. TTO equals GAPS with $C_1^v = C_4^v = \ldots = C_{2m+1}^v = \emptyset$ for each voter v. BTO refers to the special case of GAPS with $C_3^v = \ldots = C_{2m+1}^v = \emptyset$ for each voter v.

Complete or empty votes (CEV) [KL05]. Each vote is either empty or complete.

Fixed Positions (FP) [BER16]. For each vote v we have a subset of candidates C^v with distinct positions in range between 1 and m assigned.

Pairwise Comparisons (PC) [KL05]. For PC (aka *partial orders*) – more or less the standard model – for each vote v there is a transitive and asymmetric subset $\Pi^v \subseteq C \times C$.

(Unique) Totally Ordered Subset of Candidates ((1)TOS) [BER16, KL05, Ch12]. In TOS, for each vote v, there is a complete ranking about a subset $C^v \subseteq C$. An important special case of TOS, 1TOS, requires that $C^v = C'$ for each voter $v \in V$.

Briskorn et al. gave a complete picture of the interrelations of these nine partial information models [BER16]. The interrelations can be visualized with a Hasse diagram, see Figure 1. For a detailed overview, motivation, and examples for these models, we refer to [BER16].

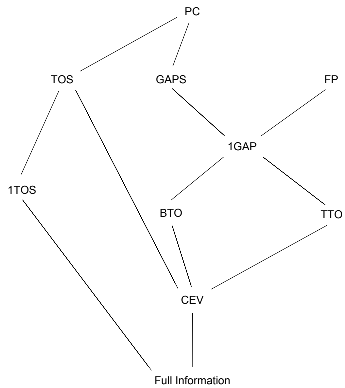

Fig. 1: Hasse diagram of the nine partial information models and full information.

3 Problem Settings

In the following, we let $\text{PIM} := \{\text{PC}, \text{GAPS}, \text{1GAP}, \text{FP}, \text{TOS}, \text{BTO}, \text{1TOS}, \text{CEV}, \text{TTO}\}$ be the set of all nine partial information models defined above and define $\overline{\text{PIM}} := \text{PIM} \cup \{\text{FI}\}$ where FI is the standard model of full information. The first problem defined in the following asks if the chair can add some new voters to the election such that c is a winner in the resulting election for at least one completion of the partial votes.

$\mathscr{E}$-(X,Y)-Possible Constructive Control by Adding Voters (PCCAV)	
Given:	An election $(C, V \cup W)$ with registered voters V according to model $X \in \overline{\text{PIM}}$, unregistered voters W according to model $Y \in \overline{\text{PIM}}$, a designated candidate $c \in C$, a non-negative integer $\ell \leq \|W\|$, and a partial profile P according to (X,Y)
Question:	Is it possible to choose a subset $W' \subseteq W$, $\|W'\| \leq \ell$ such that c is a winner of the election $(C, V \cup W')$ under $\mathscr{E}$ for at least one complete profile $P' \in I(P)$?

This way, one could combine all models of full/partial information. Our complexity analysis however concerns only the problems (FI,X), (X,FI) and (X,X) for $X \in$ PIM. (FI,FI) refers to both V and W representing full information which has been widely studied up to now and which is a special case of the other three problems. E.g., (FI,X)-PCCAV equals possible constructive control by adding voters where all registered votes are complete and all unregistered votes are partial according to X. The other problem studied asks if the chair can delete some votes such that c is a winner for at least one completion of the (remaining) partial profile, i.e., a possible winner for the residual partial election.

$\mathscr{E}$-X-POSSIBLE CONSTRUCTIVE CONTROL BY DELETING VOTERS (PCCDV)	
Given:	An election (C,V), a designated candidate $c \in C$, a non-negative integer $\ell \leq \|V\|$, and a partial profile P according to X.
Question:	Is it possible to choose a $V' \subseteq V$, $\|V \setminus V'\| \leq \ell$ such that c is a winner of the election (C,V') under $\mathscr{E}$ for at least one complete profile $P' \in I(P)$?

We obtain the destructive versions by replacing *a winner* by *not a winner*.

4 Results for Possible Constructive Control

In this section, we study the complexity of PCCAV and PCCDV. The question arises if there are problems for which possible winner and control are in P, but their hybridization is hard.

	FI	GAPS	FP	TOS	PC	CEV	1TOS	TTO	BTO	1GAP
Plurality	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Approval	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
3-Approval	P	**P**	**P**		**NPc**	**P**	**P**	**P**	**P**	**P**
4-Approval	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*
Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Veto	P	**P**	**P**		**NPc**	**P**	**P**	**P**	**P**	**P**
3-Veto	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*

Tab. 1: (FI,X)-PCCAV

	FI	GAPS	FP	TOS	PC	CEV	1TOS	TTO	BTO	1GAP
Plurality	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Approval	P	**P**	**P**		*NPc.*	**P**	**P**	**P**	**P**	**P**
3-Approval	P	**P**	**P**	*NPc*	*NPc*	**P**	*NPc*	**P**	**P**	**P**
4-Approval	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*
Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Veto	P	**P**	**P**		*NPc*	**P**	**P**	**P**	**P**	**P**
3-Veto	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*

Tab. 2: (X,FI)-PCCAV/ (X,X)-PCCAV

Theorem 4.1 *The complexity results are given as follows. Table 1 gives an overview over the results for* (FI,X)-PCCAV *(i.e., complete registered voters and unregistered voters partial according to model* $X \in$ *PIM). The results both for* (X,FI)-PCCAV *and* (X,X)-PCCAV *(i.e., for partial* V*, complete* W *respectively* V *and* W *partial according to the*

	FI	GAPS	FP	TOS	PC	CEV	1TOS	TTO	BTO	1GAP
Plurality	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Approval	P	**P**	**P**		*N*Pc	**P**	**P**	**P**	**P**	**P**
3-Approval	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*
Veto	P	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**	**P**
2-Veto	P	**P**	**P**		*N*Pc	**P**	**P**	**P**	**P**	**P**
3-Veto	P	**P**	**P**		*N*Pc	**P**	**P**	**P**	**P**	**P**
4-Veto	NPc	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*	*NPc*

Tab. 3: X-PCCDV

*same model $X \in$ PIM) are summarized by Table 2. Table 3 displays the results for X-*PCCDV *for $X \in$ PIM.*

We leave out all proofs due to space restrictions. In all tables of results, hardness entries in italic directly follow from hardness results for control under full information [Li11] or the respective possible winner problem. In particular, for the hardness result for 3-Approval-1TOS-Possible Winner, we refer to [Ch12] (the TOS result follows immediately as TOS is a generalization of 1TOS). Moreover, we know that k-Approval-Possible Winner ($k \geq 2$) [XC11] and k-Veto-Possible Winner ($k \geq 2$) are hard [BD10]. Results in boldface are new. Column FI displays the results of control under full information following from [BTT92, Li11].

We point out that Plurality and Veto are the only voting rules yielding only easiness results. 2-Approval preserves polynomial-time decidability for PCCAV given complete registered voters and unregistered voters according to arbitrary partial model.

In contrast to (necessary) control in [Re16], PCCAV for partial registered voters and complete unregistered voters is never easier than for partial unregistered voters and complete registered voters. On the contrary, we have found two problems where the problem with complete registered voters and incomplete unregistered voters is even easier than with incomplete registered voters and complete unregistered voters. As an example, 3-Approval-(FI,1TOS)-PCCAV is in P and 3-Approval-(1TOS,FI)-PCCAV is NP-complete. We further mention 2-Approval-(FI,PC)-PCCAV which is in P whereas 2-Approval-(PC,FI)-PCCAV is hard. From [Re16], we even know that the corresponding problem (for necessary control) is in P for (PC,FI) and hard for (FI,PC).

Interestingly, possible winner for a given voting rule may be hard whereas the corresponding possible control problem is easy: As mentioned, 2-Approval-(FI,PC)-PCCAV is in P, but 2-Approval-PC-Possible Winner is NP-complete [XC11]. Likewise, 3-Approval-(FI,1TOS)-PCCAV is in P, but 3-Approval-1TOS-Possible Winner is NP-complete [Ch12]. The reason for this paradox is that only the problems X-PCCDV, (X,FI)-PCCAV and (X,X)-CCAV reduce to X-Possible Winner, but not (FI,X)-CCAV.

We obtain only two hardness results with non-trivial reductions. Nevertheless, in both cases, hardness for possible winner almost directly implies hardness for PCCAV, i.e., for the proofs for PCCAV, we can embed the original hardness proofs for possible winner. This way 3-Approval-(FI,PC)-PCCAV is hard due to the possible winner hardness for 2(!)-

Approval [XC11], and 2-Veto-(FI,PC)-PCCAV is hard due to the hard possible winner problem for 2-Veto [BD10]. (On the contrary, given partial registered voters, CCAV in 2-Veto is hard under the PC model reducing directly from 2-Veto-POSSIBLE WINNER.)

We point out that six models (GAPS, FP, 1GAP, BTO, TTO and CEV) never increase the complexity of possible control whenever the corresponding standard control problem and possible winner are easy. Note that GAPS and FP are two general partial models and our initial expectation was that – similar to PC – they yield many hardness results.

5 Results for Possible Destructive Control

In contrast to constructive control, we achieve a global P result for all scoring rules for the destructive version both for PCCAV and PCCDV. The main idea is to check for any model if there is a non-distinguished candidate d beating the despised candidate c for at least one extension. Roughly speaking, we rank d as high as possible and c as low as possible in each vote and derive control problems under full information concerning these two candidates.

6 Conclusion

We studied the complexity of PCCAV and PCCDV in k-Approval and -Veto under nine different models of partial information. Surprisingly, there are only two hardness entries not directly reduced from possible winner or control under full information (although we can almost directly adopt the possible winner proofs). We further have that – in contrast to necessary control in [Re16] – CCAV with complete registered voters and partial unregistered voters is never harder than the other way around. For two instances, (X,FI) is really harder than (FI,X), namely for 2-Approval-PCCAV (PC) and 3-Approval-PCCAV (1TOS). Note also that particularly for 2-Approval-CCAV, (PC,FI) is easier than (FI,PC) for necessary control, but harder for PCCAV. For destructive control, we could settle a polynomial-time result holding for all scoring rules. Open questions for future research include the open problems in this paper or the extension of our study to other voting rules or control types.

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